Synthesising networks

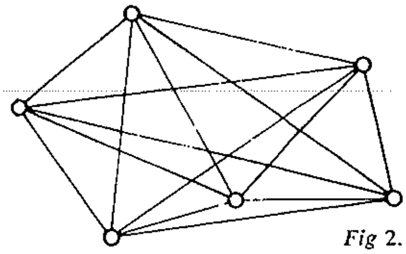

Fig 2.

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In its most basic form a network is formed from a set of points (nodes or vertices), some pairs of which are joined together by lines (links or edges). Historically many transport networks have been evolved on a piecemeal basis. Today some planning techniques require an alternative method for constructing networks. Generally in any given area there are important points (often called origin nodes) which must be served by a network. Here consideration will be given to physical networks (e.g. transport networks), although the methodology employed can be adapted for other kinds of networks.

In transportation engineering the problem of network design has yet to be satisfactorily solved; this is particularly important when planning for the new town situation. Even in existing towns, for a given land use distribution pattern there is as yet no systematic method for network design.

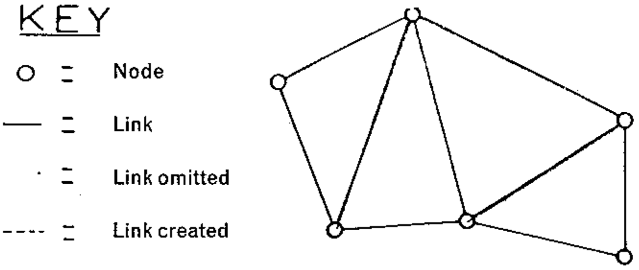

Fig 1.

A partial solution for test purposes is the spider network (Fig 1), which consists of the origin nodes in an area (often representing traffic zones) joined to immediately adjacent nodes by direct links. A spider network can be tested in comparison with existing or planned networks, but because of its nature is usually impractical to implement. Other methods have been evolved for designing networks between origin nodes, notably the soap film method of Miehle[1] and the Wye-Delta Transformations of Akers[2]. Grids have also been used for transport network design, e.g. the square grid used in American cities and, in this country, in central Glasgow and the new city of Milton Keynes. Other grid shapes have been proposed for transport networks, particularly the hexagonal grid[3]. The main theoretical advantage of grid networks is the ease in which they can be analysed and simulated, particularly for studies of uniform areas[4, 5, 6]. In practice few areas are truly uniform, and so the theoretical basis which supports a grid network is eroded.

As part of a study of network characteristics begun at the University of Strathclyde, an algorithm was written in Algol 60[7] to synthesise networks between origin nodes by strictly geometric principles. These nodes may be randomly distributed and serve non-uniform areas. The starting basis of this algorithm is the assumption that there is always a potential demand for travel (or some other measure of inter-nodal service) between all possible combinations of origin nodes. This potential demand can be represented by a desire line network (Fig 2), where all origin nodes are interconnected by straight links. The algorithm synthesises a network by manipulating and simplifying the desire line network between the origin nodes. This manipulation and simplification is based on geometric considerations and the resulting network depends on the interaction of two control variables. These variables are: (a) δ, the minimum inter-nodal distance (Fig 3); and (b) θ, the minimum inter-link angle at a node (Fig 4).

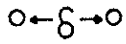

Fig 3.

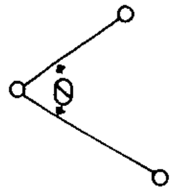

Fig 4.

The choice of δ and θ as control variables in the algorithm was made from an investigation[8] into the parameters which control the shape of geometric networks. A network's main components are its nodes and the links which interconnect certain pairs of nodes. There are basically four ways in which the shape of such a geometric network can be changed. Two ways are boolean operators, namely the existence or non-existence of a node or link. Two are continuous, the inter-nodal distance and the angle between links at nodes. The inter-nodal distance controls the density of nodes in the area served by the network, while the inter-link angle controls the density and duplication of links in the network.

The algorithm has been written in five sections:

(1) Input of the co-ordinates of the origin nodes in order and values for the control variables. The control variables include δ and θ as outlined above and parameters to direct the operation of the algorithm. The final network can be output either after a specified number of iterations through the algorithm or else when a condition of network stability is reached. The algorithm begins by creating a desire line network between the origin nodes (as in Fig 2).

(2) Amalgamation of links close together. Each node is taken in turn and the links radiating from it found. The angles between the links are calculated and compared to the input value of θ. If the smallest angle is less than θ, then the two links which include this angle are considered to be the two sides of a triangle. The links are amalgamated using the solution to the 'Three Villages Problem'[9] according to Fig 5. When all the interlink angles at one node are greater than θ, the next node is considered.

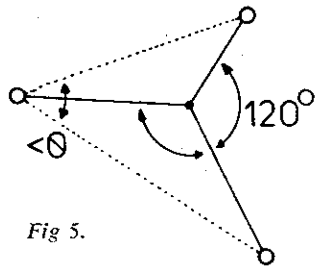

Fig 5.

(3) Creation of nodes at the intersection of links. Each link is examined in turn and compared to all the other links. If an intersection between links is detected a new node is created at the intersection point, provided this is further than distance δ from the existing nodes. If the intersection point is less than δ, then the arrangement of the links is readjusted (see Fig 6).

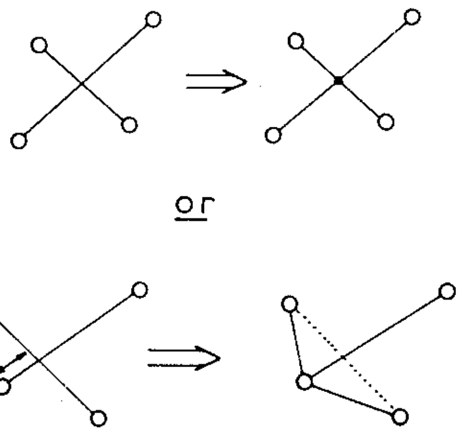

Fig 6.

(4) Amalgamation of nodes close together. All the nodes are considered in turn. The distance separating each pair of nodes is compared to the value of δ. If this distance is less than δ, then the nodes are amalgamated according to the following steps:

(a) If one of the pair of nodes is an original input node, then the other is superimposed on it and all connecting links are readjusted.

(b) Otherwise the midpoint of the pair of nodes is taken as the position of the amalgamated node.

These steps are shown in Fig 7.

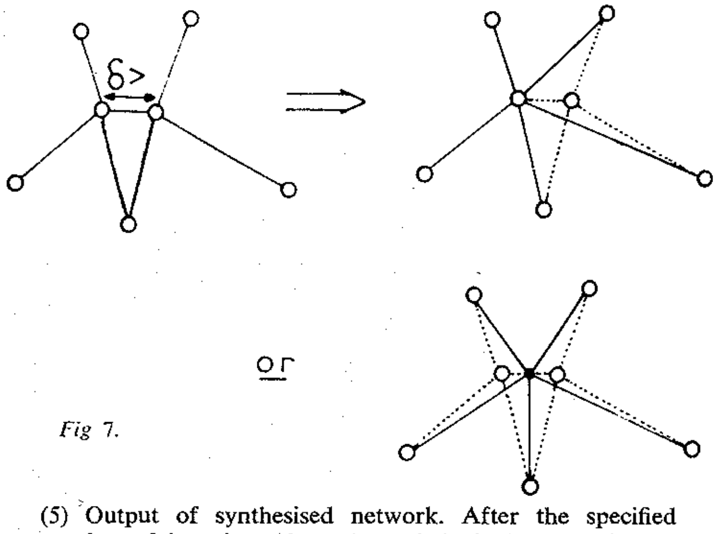

Fig 7.

(5) Output of synthesised network. After the specified number of iterations through sections 2, 3 and 4 (which need not be used in this order), the algorithm outputs the network as a list of nodes and inter-connecting links. This output may be used to control a graph plotting algorithm or else a manual drawing of the synthesised network may be made.

As an example of the potential of this algorithm, the origin nodes used in Figs 1 and 2 were input together with several values for the control variables δ and θ. These networks are shown in Fig 8, which demonstrates that the algorithm will synthesise different networks from the same set of origin nodes dependent only upon the input values of δ and θ. The algorithm can be used to generate various network shapes to serve a given set of origin nodes. Each synthesised network can then be tested and the one which gives the best performance chosen for further work.

Present limitations of the algorithm

The main limitation of this algorithm is the computational time required. From experience with the University of Strathclyde ICL 1905 computer, a network synthesis between 30 origin nodes took about four hours. However, the same synthesis takes only 10 minutes on the University of Newcastle upon Tyne IBM 360/67. Little difference has been observed in the computational time requirements between the original Algol 60 version and a translation into Fortran IV. The computational time rises geometrically with the number of origin nodes. Therefore it may be impossible at present to synthesise a network between more than 200 origin nodes.

A second limitation is a present inability to detect node clusters. Ideally all the nodes should be considered simultaneously and those which form clusters identified. Then the algorithm should operate on the clusters from the densest to the least dense.

A third limitation concerns the inability of the algorithm to exclude parts of an area served by the origin nodes, e.g. physical barriers which may be uneconomic to cross. Obviously this is unimportant in an area with no such barriers. However, when there are barriers, a manual adjustment will be necessary to a synthesised network.

A fourth limitation, which combines features of the others above, is that the algorithm may not necessarily synthesise a symmetrical network between a symmetric set of origin nodes. When the network asymmetry is small, the limitation will be unimportant.

Conclusion

An algorithm has been reported which can be used to synthesise networks between any given set of origin nodes. This algorithm may be useful for network design in transportation studies. Given the origin nodes of an area, several networks can be synthesised. These networks not relying on existing infrastructure would be useful for comparative purposes. From a set of synthesised networks the one which gives the best performance can be used for further network efficiency analysis. The systematic approach of this algorithm allows reproducible results to be used in a study of network efficiency and shape criteria. This is particularly valuable in designing for new transport networks and allows the final network to reflect the service requirements of the origin nodes.

ACKNOWLEDGMENT

To the Traffic and Highway Engineering Section of the Department of Civil Engineering at the University of Strathclyde, for the facilities used in undertaking part of this research, and to the Science Research Council for support since January 1, 1972.

REFERENCES

[1]Miehle, W. Link-length minimisation in networks. *Operations Research*, 6, 232.

[2]Akers, S. B. The use of Wye-Delta transformations in network simplification. Rand Symposium on Mathematical Programming, Santa Monica, Calif., March 1956.

[3]Buchanan, C. D. *et al.* Traffic in Towns, p. 43. HMSO, London, 1963.

[4]Holroyd, E. M. The optimum bus service: a theoretical model for a large uniform urban area. *Proc.*, Third International Symposium on the Theory of Road Traffic Flow. New York, 1965.

[5]Wilshaw, V. Optimum road networks in idealised urban areas. MSc Thesis, University of Liverpool, 1965.

[6]Tanner, J. C. and S. V. Magee. A theoretical comparison of triangular motorway networks with other idealised forms. RRL Report LR359. Road Research Laboratory, Crowthorne.

[7]Lesley, L. J. S. A study of public transport in relation to urban form. PhD Thesis, Appendix 6.6.1. University of Strathclyde, 1971.

[8]Lesley, L. J. S. *Ibid*, Chapter 5.

[9]Pedoe, D. Circles. Pergamon, Oxford, 1957.

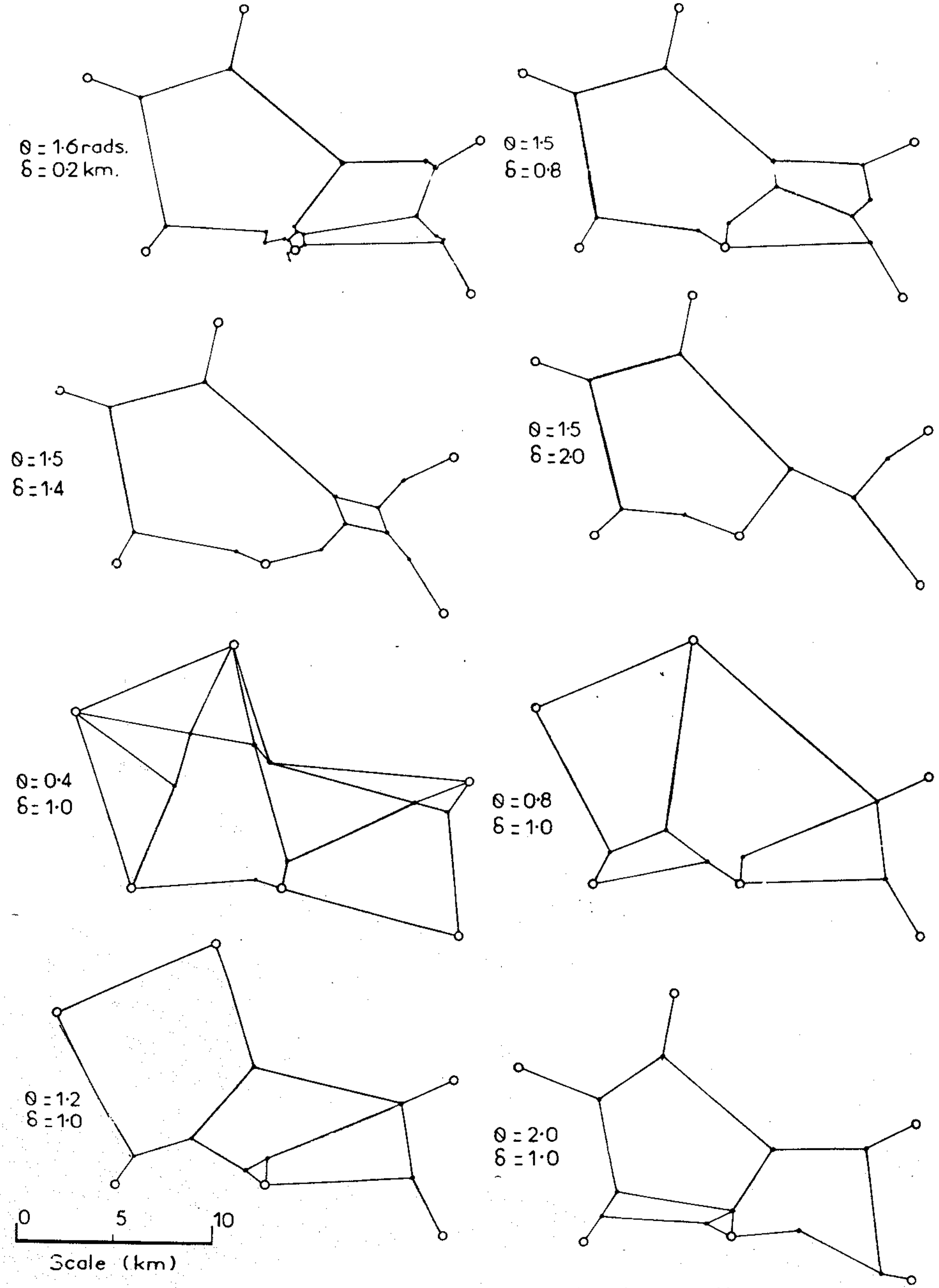

Fig 8. Eight networks synthesised between the origin nodes used in Figs 1 and 2.