Urban Railway Capacity in Peak Periods

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ABSTRACT

In previous research on urban rail operations, capacities have been calculated using fully deterministic procedures based on the calculation of the minimum time interval between successive arrivals at a station. This paper adds to previous contributions on the analysis of traffic flow of railway systems in that a stochastic model for the passage of trains is derived in terms of queueing theory. Statistical fluctuations in station stop times occur in urban commuter railways at bottleneck stations and furthermore some randomness is present in the time headways of trains passing the entry point to the station. Thus it is appropriate to extend the deterministic models by considering a railway block signalling section (from the outer home signal to the station starter) as the service component of a classical single server queue. Various time headway distributions are considered as inputs to the station approach and particular attention is given to a Semi-Poisson model of headway, similar to certain models used for analysing road traffic headways.

The work is valid for a general class of distributions in both input and service and admits of wide application in modelling railway signalling systems. As an example data collected from a study of London Transport Underground Railways were used to calibrate headway models and then applied to model a typical block section including a station stop. It is believed that a queue-theoretic basis for mean waiting time in the system and mean queue length for a given traffic intensity provides a better approach to the planning of levels of service than the present operating procedure of some railway bodies who calculate the capacity under the deterministic minimum time headway criterion and then operate at some proportion of that level.

1. INTRODUCTION

A major difficulty in urban railway operation is the satisfactory definition of capacity for train arrivals at a station entrance in a commuter railway system.

Traffic flow theory for railway systems has involved the development of theoretical models for the minimum time interval between consecutive arrivals at an idealized railway station[1], and capacity calculations have been based on fully deterministic models assuming fixed value parameters. Lang and Soberman[2] discuss various concepts of capacity for certain signalling regimes, in particular they derive expressions for the capacity of a block signalling system with and without a station stop, as a function of approach speed. A more detailed consideration of the principles of trackside signalling is found in Lagershausen[3], who extends the work from manually driven systems on fixed block operation to running at "electric sighting distance", which is an automated system with a theoretically continuous monitor of headway of the train in front. It is well known that closer headways can be obtained by increasing the number of trackside signals until in the limit intermittent signalling becomes continuous; Askew et al[4] give a practical illustration of this. The concept of running at an ideal minimum safe headway in the presence of imperfect information is examined by Rahimi et al[5], who give an analytic expression for a general safety margin which would enable a practical realisation of the definition proposed by Bergmann[1] that a railway system is based on the operational philosophy that successive vehicles be separated by a gap which is not less than the instantaneous stopping distance of the following vehicle. Morimura[6] discusses a practical study of line capacity from a deterministic view, whilst Yamada proposes a random variable approach[7].

This paper adds to previous work on traffic flow in railway systems in that a stochastic model is introduced to model the signalling system in queueing theory terms. In the notation of Kendall[8], this is (GI/G/1). The flow of trains through a single track station is essentially a

stochastic process with the service mechanism of the queue being dependant on the form of signalling employed on the station approach. The presence of random variation in driver acceleration and deceleration characteristics, together with the possibly substantial variation in station stop time for loading and unloading, constitute the service time distribution. Likewise variations in the input stream from timetabled (usually equally spaced) time headways are also accommodated by a general input distribution. A significant feature of a close headway passenger service is the nature of the time headway distribution which can loosely be described as having two components, a 'free' and a 'constrained' component. A Semi-Poisson[9] model of time headway is proposed and its suitability (see fig.3) is discussed for a railway model.

2. FORMULATION OF CAPACITY PROBLEM

The underlying philosophy of a block signalling system is that the section in which a train is travelling should at all times be protected from rear-end collisions. In the event of a signal being passed at danger some automatic device applies the emergency brakes and the train is halted in the emergency braking distance, thus a train in any particular section of track is separated from an oncoming train by at least an emergency braking distance. This safety factor is the basis for the design of conventional railway systems. The principles of operation are described here for the case of an idealized station approach. (Fig 1 shows a schematic representation of the time and velocity trajectories in a station approach).

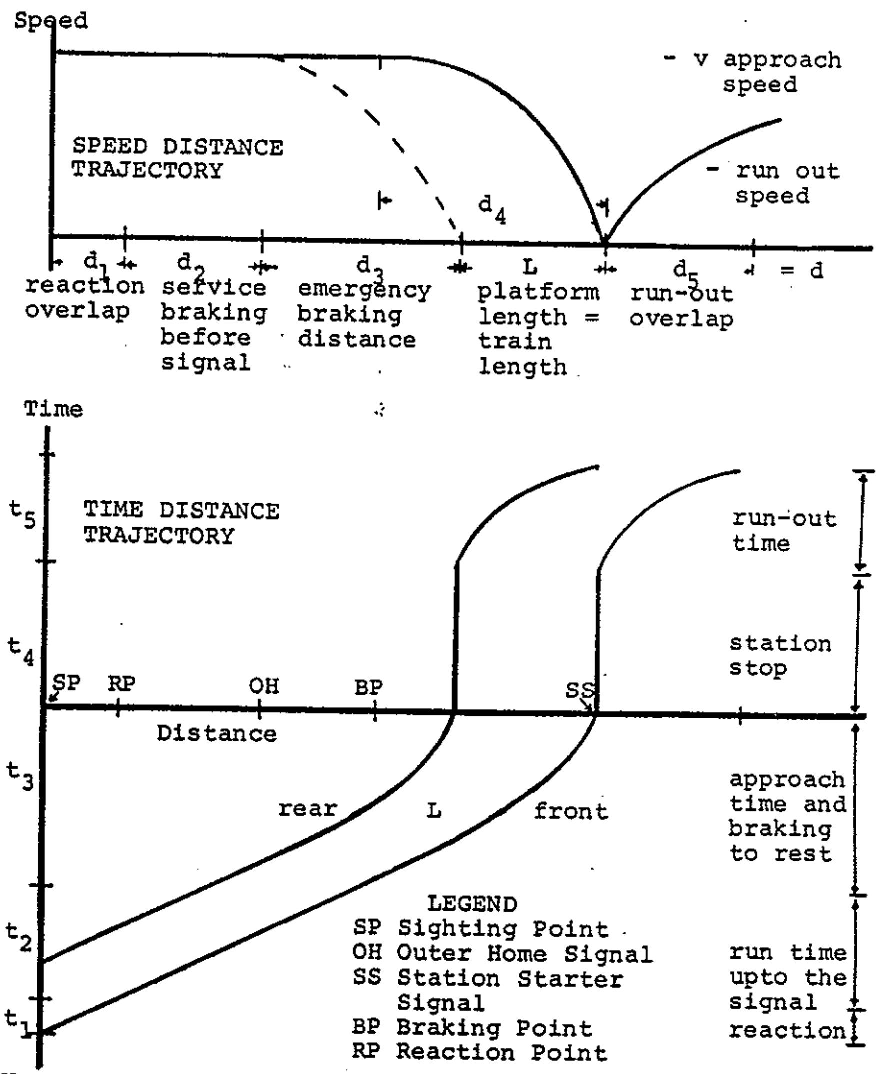
2.1 Description of station approach

Consider a sequence of trains all of which stop in the station as represented in fig 1. Let the n th train approach at velocity v and consider the velocity trajectory at various distances from the sighting point of the outer home signal. In section 2.2 the elapsed time from this point is derived. The approaching train is assumed to be making an unchecked approach to the station. Thus it runs at v over the reaction distance d_1 and over the service braking distance d_2. It has now reached the outer home signal which it passes. (If the signal were at danger then the emergency brakes would have been applied, braking the train to rest in distance d_3). The train now brakes to a halt by applying the service brakes over a distance d_4. The train comes to rest in the platform berth of length L which is displaced d_3 from the outer home signal thereby ensuring that whilst the train is stood in the platform it is protected from a rear-end collision should the n+1 th train attempt to enter the section. The n th train waits till the passenger loading and unloading process is complete (a process which varies especially between surface and underground systems depending on how the doors are controlled). When the doors are all closed and when this has been proved, the train is free to run out provided that the station starter signal permits this. When the train has covered a distance at least equal to its length the outer home signal clears to allow the next train entry to the platform. The run-out distance d_5 is caculated to prevent a rear-end collision beyond the starter overlap.

2.2 Conventional approach to minimum time headway

The sequence of operations described in section 2.1 in distance terms is more usually considered in terms of the time components and thus an expression for the total time h taken to pass through the station section is obtained. The presentation is simplified if we consider a railway system which consists of equi-length trains with identical braking and acceleration characteristics. If this is not true then the safety aspects of the problem require that the system be safe in all cases and thus the poorest

Fig. 1 SCHEMATIC TIME AND DISTANCE HEADWAYS DURING A STATION APPROACH



Note acceleration and deceleration rates assumed constant
Broken line shows emergency braking trajectory.

performance of the longest train is the limiting factor. Hence a non-trivial consequence for the operation of the line at maximum capacity is that different types and lengths of train stock should not be used in the same service.

2.3 Notation for minimum headway

- h = Minimum time headway based on the maintenance of a safe stopping distance between successive vehicles
- t_1= Combined driver and system response time
- t_2= Run-time over sighting distance to outer home signal
- t_3= Time taken from passing signal to braking to rest
- t_4= Station stop time (wheels stop to wheels start)
- t_5= Run-out time to clear outer home signal

Thus we define the conventional minimum time headway as

$$h = t_1 + t_2 + t_3 + t_4 + t_5 \qquad (1)$$

Whilst the distance headway d is equivalently given by

$$d = d_1 + d_2 + d_3 + L + d_5 \qquad (2)$$

A particularly simple set of relationships can be derived for t_2, t_3 and t_5 in terms of the approach speed v, the service braking rate f_1, the emergency braking rate f_2 and the acceleration rate a_1 (all assumed constant). Bergmann[1] quotes the following result:

$$h = t_1 + \tfrac{1}{2}v(f_1^{-1} + f_2^{-1} + a_1^{-1}) + t_4 + L/v \qquad (3)$$

subject to certain restrictions on L or v. Thus a minimum time headway equation can be derived in terms of v the approach speed. This equation or a more refined version if the signalling incorporates intermediate home signals is the basis of the signal engineers capacity calculation since q, the flow per unit time, is inversely related to the minimum headway (q=k/h in appropriate units k).

The weakness of this approach is that whilst a theoretical lower bound exists on h for all v, thus implying an upper bound on q; no allowance is made for the presence of randomness which reduces the capacity over a fixed period

of time corresponding to a peak period of commuter travel when considerable random variations in (3) occur.

3. QUEUEING THEORY INTERPRETATION OF BLOCK SIGNALLING

It is possible to conceive of a railway block section as the service component of a classical single server queue with a 'first come first served' priority rule. The approach here adopted is based on equation (3) so it is convenient to define as T_n the epoch of arrival of the n th train at the sighting point of the outer home signal. Let U_n be the service time of train n, thus at epoch T_n+U_n the next train may enter the section. (The service completion corresponds with the outer home signal changing from danger to clear). Should the (n+1)th train arrive at the sighting point before epoch (T_n+U_n), at epoch T_{n+1}, then it brakes to a halt in front of the outer home signal. It is important to note that the service mechanism is not timed from passing the outer home signal (which could be an imaginary stop line); by adopting this device, the service mechanism of the queue exactly parallels the minimum time headway equation.

3.1 Derivation of expression for queueing time

Let us denote by X_n the queueing time of train n and by Z_n the time between arrivals of n and (n+1)th train at the sighting point of the outer home signal. (see fig. 2) It is assumed that U_n the service interval is independent of Z_n the inter-arrival time. Let A(x) and B(x) be the distribution functions of the inter-arrival and service time respectively. It is assumed that a stationary distribution of queueing time exists, so we proceed by considering the relationship between the queueing times for the n and (n+1)th train, X_n and X_{n+1}, i.e.

$$X_{n+1} = \mathrm{Max}(X_n + U_n - Z_n\ ,\ 0) \qquad (4)$$

Let the stationary distribution of queueing time be F(x), We define an intermediate distribution K(x) for $U_n - Z_n$, so

$$K(x) = \int_0^\infty B(x+z)\,dA(z) \qquad (5)$$

Since X_n and $U_n - Z_n$ are independent, the distribution function of $F(x)$ satisfies the well known integral equation of the Wiener-Hopf type.

$$F(x) = \int_0^\infty K(x-y)\,dF(y) \qquad (6)$$

3.2 Modifications to queueing time in a signalling system

The usual equation of queueing theory (4) requires careful interpretation in application to a railway block signalling system. Conventional queueing theory assumes that service may begin at the epoch when the previous service had been completed, that is to say there is no 'connection delay', before service begins. If there were a connection delay C_n, the queueing time becomes

$$X_{n+1} = \mathrm{Max}(\, X_n + U_n - Z_n + C_n,\ 0) \qquad (7)$$

where the distribution of C_n is required, thus further complicating the integral equation.

The approach of section 3. in separating the timing point from the imaginary stop-line has introduced a connection type phenomenon involving a simple connection delay or a process loosely described as 'expedited connection'. The situation is illustrated in fig. 2, and described in section 3.3. Thus to complete the discussion of queueing time we note that a train is considered to be in a queue when it is unable to enter the station and is thereby incurring a time penalty in braking, at rest, or in reacceleration. The modified queueing time equations are derived in the Appendix.

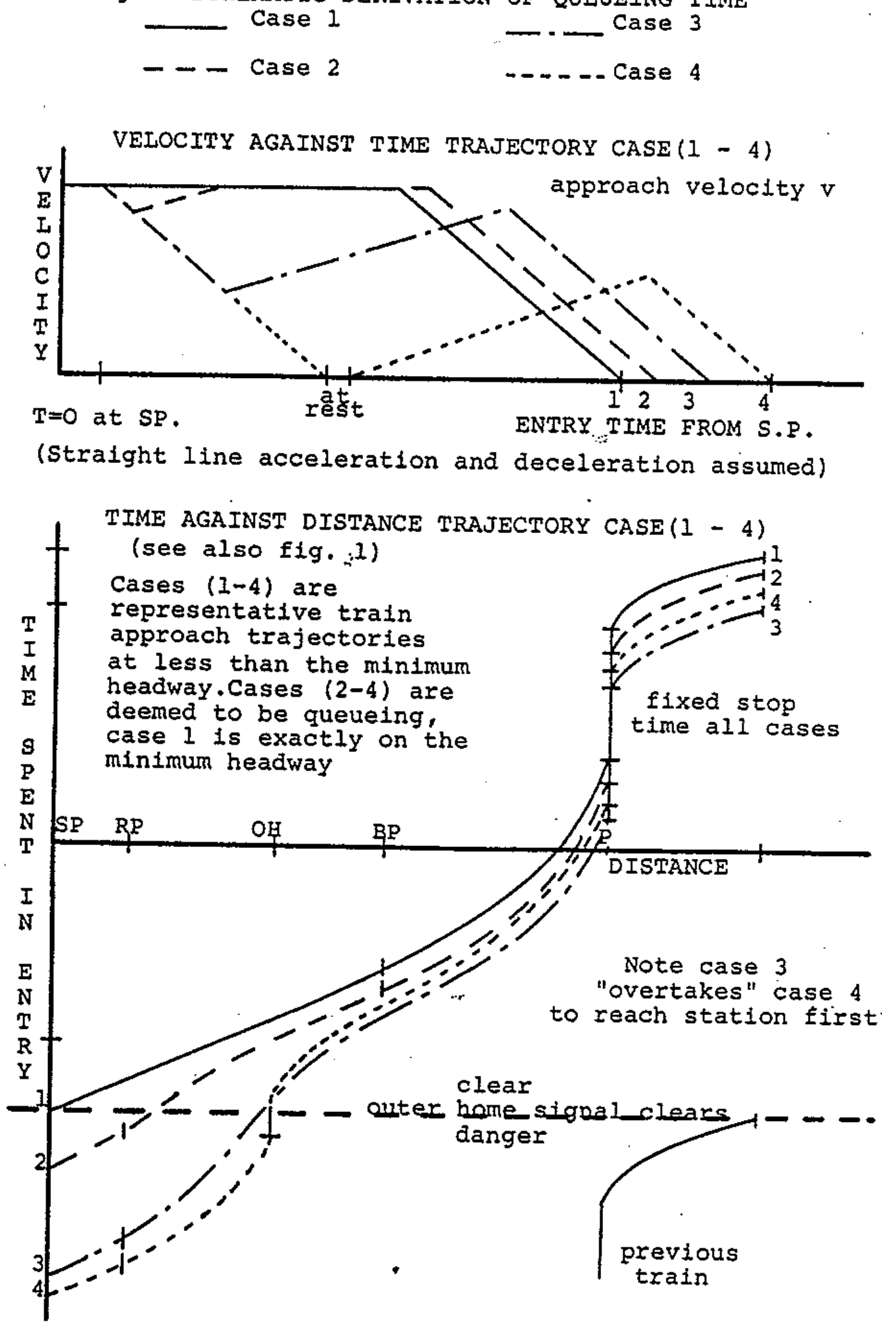
3.3 Types of delay to oncoming trains

(The categories correspond to the train trajectories fig 2)

3.3.1 Case 1 No delay to oncoming train, since the signal is clear at epoch of arrival.

3.3.2 Case 2 The train is checked for a short interval but is able to reaccelerate and achieve the usual approach

Fig. 2 SCHEMATIC DERIVATION OF QUEUEING TIME



velocity v, as given in section 2, at the braking point.

3.3.3 Case 3 The train is not brought to rest before the outer home signal clears, but it is sufficiently checked that it cannot regain speed v before the braking point.

3.3.4 Case 4 The train is brought to rest at the outer home signal and waits on the stop line until the signal clears. It then accelerates to a new braking point and decelerates to stop in the platform.

Both case 3 and case 4 require considerable driver expertise in determining when to brake for the run-in after the period of acceleration. In practice drivers coast to make sure of not over-shooting the stop point in the platform. (This is analogous to the problem of covering a given distance in minimum time, see lemma 1, Appendix)

3.4 Practical application of simple queueing results

From the Appendix the expressions for queueing times have been derived in the 'expedited connection' case as:

$$X_{n+1} = K_1(X_n+U_n-Z_n)^2 \qquad (8)$$

whilst in cases 3 and 4, the connection delay is

$$X_{n+1} = (X_n+U_n-Z_n) + K_2 \qquad (9); \quad \ldots.+K_2' \quad (9a)$$

where K_1, K_2 & K_2' are expressions involving the approach speed and deceleration rate, which we have postulated to be random variables, reflecting the driver characteristics. Even if we assume K_1, K_2 & K_2' to be constants the modified integral equations are not to the author's knowledge soluble in the general case of a (GI/G/1) system adjusted for both a connection delay and an expedited connection. The procedure here adopted is initally to present a simple model using equation 4 with the understanding that in the case of very short intervals of delay, the simple model over-estimates the true value of delay, whilst for longer intervals it depends on the values of K_2 and K_2'.

Essentially a railway block signalling section is a single server queue fed by and feeding another single server queue. It is thus in the spirit of this paper to allow the general formulation as (GI/G/1) and interpret queue lengths of one, two or three as being trains effectively waiting at the outer home signal, but in block signalling sections of their own. The occurrence of long queues (in railway terms two or more is long) is a design feature which should not be permitted as it is likely that because of connection type delays, the move-up time will be considerable for the second or third in the queue. In this case the simple waiting time formula definitely under-estimates the actual delay. The possibility of restricted output from the queue caused by the occupancy of block signalling sections in front of the starter signal would cause a train to be held in the platform for longer than the necessary passenger loading and unloading cycle. This effect is really a consequence of a series of queues interacting with each other; such problems of restricted waiting space tend to feature in urban railways with short inter-station distances (½ mile or less).

In assessing the operation of a station approach at different levels of traffic intensity the following entities are easily derived from the simple queueing model based on equation 4 (subject to the limitations outlined on the mathematics and the waiting space).

(I) Mean waiting time (queueing + service time) for a train to pass through the station section.

(II) Mean queue length of trains at the outer home signal, including those assumed to be queueing there.

(III) The probability distribution of queue length at epochs of arrival.

It is believed that these three concepts should play an important role in capacity considerations.

4. MODELLING THE INTER-ARRIVAL HEADWAY DISTRIBUTION

Various theoretical models of time headway for vehicular traffic have been proposed and it is instructive to consider whether road traffic headway models may be used in the field of urban railway operation. This is because a close headway passenger service resembles the more familiar single-lane road traffic flow. Special attention has been given to the Semi-Poisson model[9] of headways which attempts to explain the zone of emptiness in front of a vehicle. This zone of emptiness has an immediate interpretation in a railway block signalling system, since it is in distance terms at least an emergency stopping distance. At any point in the system the time headways are such as to incorporate a minimum time interval consistent with safe passage. Thus for the station approach of section 3 the minimum time headway for train passage through the station defines the safety zone of emptiness. At other points between stations the minimum time headway is less since there is no stop time component. It is as a consequence of this that the station approach is always the bottleneck. The Semi-Poisson model has the attractive theoretical feature of modelling the headways which are constrained by the minimum safe headway, whilst describing the free component by the Poisson process.

4.1 Some practical results

From a study of time headways conducted over a five week period in the morning peak on a London Transport Underground Line, it appears that a displaced log-normal distribution fits the data best, but that the Semi-Poisson model also fits adequately, (see fig 3). The fitting procedure follows Buckley[9]. The advantage of the displaced log-normal is that it directly estimates the minimum time headway possible which could of course be calculated and used as an input to the headway model. On balance the Semi-Poisson is preferred although it does not incorporate a cut-off value for the minimum headway

possible. Another possibility is the Hyper-Erlang headway model of Dawson[10], although this has not been tested. A final remark concerns the sample cumulative frequency function which exhibits on log-scale (fig 3) a very pronounced linear tail. This is interpreted to mean that the trains travel in bunches with the time headway between bunches having a negative exponential distribution. This is the random queues model of Miller[11], a very suitable hypothesis for underground railway traffic.

4.2 Time headway models

Several of the time headway models mentioned in 4.1 are defined here in their probability density form.

4.2.1 Three parameter displaced models

Displaced Gamma/Erlang

$$(\beta\Gamma(\kappa))^{-1}((t-\alpha)/\beta)^{\kappa-1}e^{-(t-\alpha)/\beta} \qquad (10)$$

Displaced Log-normal

$$(t-\alpha)^{-1}(2\pi\sigma^2)^{-\frac{1}{2}}\exp(-\tfrac{1}{2}(\log(t-\alpha)-\mu)^2)/\sigma^2) \qquad (11)$$

4.2.2 Semi-Poisson models (four parameters)

Gamma zone of emptiness

$$(\phi\{\beta\Gamma(\kappa)\}^{-1}\{t/\beta\}^{\kappa-1}e^{-t/\beta} + (1-\phi)(\beta\Delta)^{-\kappa}\lambda e^{-\lambda t}\int_0^t\{\beta\Gamma(\kappa)\}^{-1}\{z/\beta\}^{\kappa-1}e^{-z/\beta}dz$$

$$\Delta = \beta/(1+\lambda\beta) \qquad (12)$$

Gaussian zone of emptiness

$$(\phi/\{2\pi\sigma^2\}^{\frac{1}{2}})\exp(-\{t-\theta\}^2/2\sigma^2) + (1-\phi)(\exp\{\theta\lambda-\tfrac{1}{2}\sigma^2\lambda^2\})\lambda e^{-\lambda t}N(t) \qquad (13)$$

$$N(t)= \int_{-\infty}^{t}\{2\pi\sigma^2\}^{-\frac{1}{2}}\exp(-\{z-\theta\}^2/2\sigma^2)dz$$

4.3 Modelling general queueing processes

A useful procedure for efficiently solving the queuing equations is here presented based on Smith[12]. If the generating functions of both arrival and service time distributions are always the reciprocal of a polynomial of finite degree then the roots of (14) give rise to a very fast procedure for calculating the steady state values of queue length distribution.

$$A^*(-s)B^*(s) = 1 \qquad (14)$$

$A^*(s)$ and $B^*(s)$ are the Laplace transforms of arrival and service time distributions respectively. As an example we note that the transform of the general Erlang distribution E_k has transform $F^*(s)$ given by:

$$F^*(s) = \prod_{1}^{k}(1 + \theta_i s)^{-1} \qquad (15)$$

which has k parameters $\theta_1 \ldots\ldots \theta_K$

If all the θ_i are equal, this is a chi-squared distribution which we denote $E_k(\chi^2)$. Smith[12] has noted the usefulness of a weighted sum of χ^2 distributions with various even number degrees of freedom in modelling widely different types of behaviour. This formulation is hence characterized by a generating function which is always the reciprocal of a polynomial and therefore applicable in (14).

This technique is exploited in section 5.3 in showing how a queueing model of a bottleneck station of a London Transport railway is constructed from data available on the service and arrival time distributions.

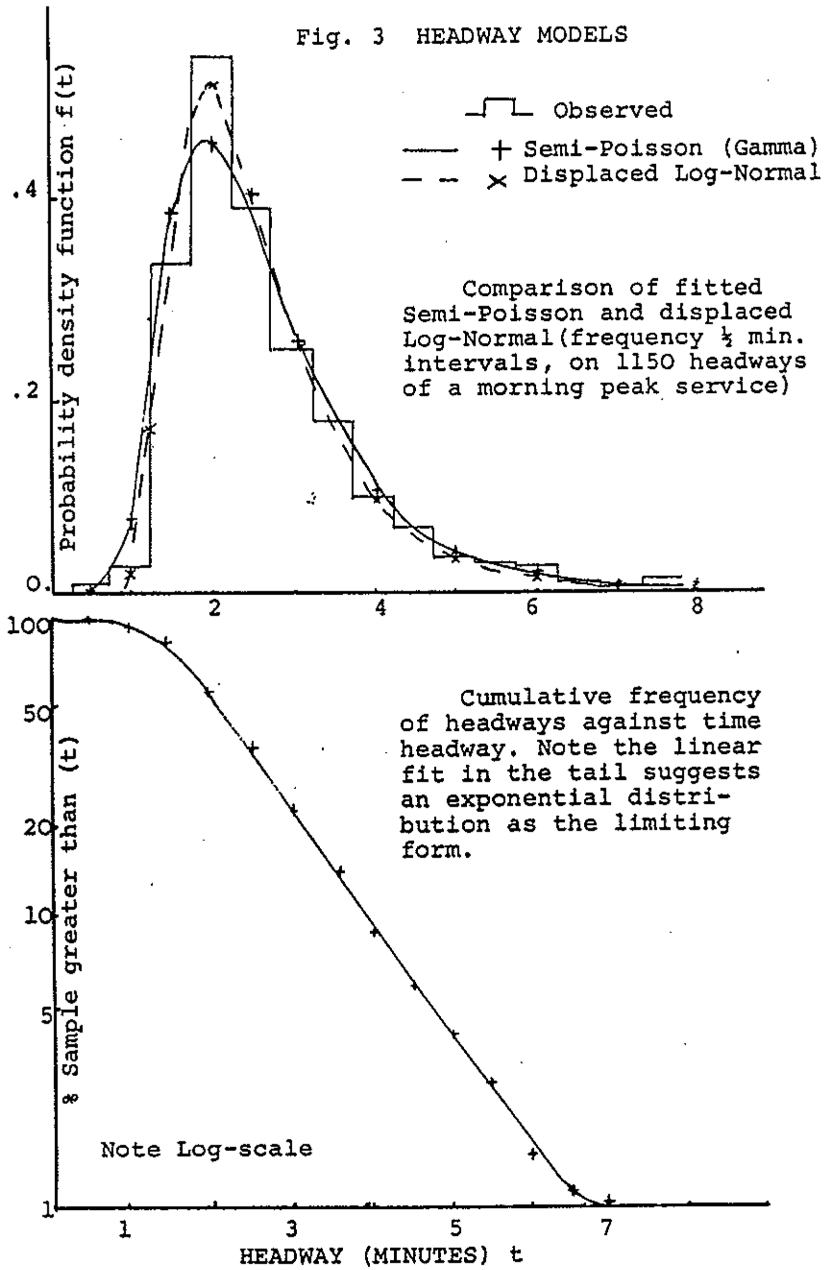
Analytic expressions for the expected waiting time, the queue length probability distribution and the expected queue length are known for the system $(GI/E_m(\chi^2)/1)$, see Wishart[13]. In fact a wider class than χ^2 is permissible in the service process by suitably adjusting the parameters. The preferred model for a railway block signalling section is a Semi-Poisson inter-arrival model with a displaced Erlang service distribution, whilst a simpler model would consist of an Erlang arrival and a χ^2 service which in the latter case can be easily computed.

5. SOME PRACTICAL RESULTS FOR MEAN DELAY AND QUEUE LENGTH

The effect of various traffic intensities on mean waiting time and queue length distribution is plotted in fig 4. It is convenient to adopt a standardised time scale by making the mean headway unity and the mean service time ρ, which is thus the traffic intensity.

5.1 Modified queueing equations

Fig. 3 HEADWAY MODELS



It is convenient to consider the Laplace transforms of theoretical arrival and service distributions of equation (14) in the notation of section 4.3 thus:

$$E_k(\chi^2)/E_m(\chi^2) \qquad (1-s/k)^k(1+\rho s/m)^m = 1 \qquad (17)$$

$$E_k/E_m/1 \qquad \prod_i(1-\theta_i s/k)\prod_j(1+\Psi_j s/m) = 1 \qquad (18)$$

$$\Sigma_i\,\theta_i = k \qquad \Sigma_j\,\Psi_j = m\rho$$

$$D/E_m(\chi^2)/1 \qquad e^{-s}(1+\rho s/m)^m = 1 \qquad (19)$$

$$E_k(\chi^2)/D/1 \qquad (1-s/k)^k e^{\rho s} = 1 \qquad (20)$$

$$SP/E_m(\chi^2)/1$$

$$\{\phi(1-\beta s)^{-k}+(1-\phi)(1-s/\lambda)^{-1}(1-\Delta s)^{-k}\}(1+\rho s/m)^m = 1 \quad (21)$$

Where D denotes regular and SP denotes Semi-Poisson.

5.2 <u>Expression for expected waiting time</u>

Smith(12) gives two expressions for the expected waiting time, if the service distribution is E_m then

$$\text{(I)} \quad E(w) = -\sum_i s_i^{-1} \qquad (22)$$

where E(w) is the expected waiting time and s_i are the roots of an equation of the form of (17)-(21) such that the real parts of s_i are negative. (If the model is $E_k/E_m/1$ then there are m such roots). If the arrival process is E_k

$$\text{(II)} \quad E(w) = \sum_j s_j^{-1} + (V_1^{-1} + V_2^{-1} + (1-\rho)^2)/2(1-\rho) \qquad (23)$$

and s_j are the roots with strictly positive real parts, and V_1, V_2 are the variances of the inter-arrival and service times respectively.

Kotiah et al(14) present a highly efficient iterative scheme for solving an equation of the form of (17) to (21) to obtain the roots for substitution in (22) or (23). Further the roots obtained when the service is χ^2 enable the procedure of Wishart(13) to be followed directly to obtain the queue length distribution.

5.3 <u>Discussion of results for mean delay</u>

Several curves representative of operating conditions likely to be found in urban railways are given in fig 4.

which have been prepared with data from the London Transport Underground Railways. When considering a peak service, it is the traffic intensity which assumes the role of the independent variable. The minimum headway equation (3) is quite adequate in defining the time taken to pass through the section at very low traffic intensities when it is very unlikely that there is a queue, but as traffic intensity rises so the queueing time rises. Fig 4.1 shows how the probability of having 0 or 1 train in the system varies with intensity and fig 4.2 shows what is happening to the queue length. We note that the effect of greater regularity in either service or arrival cuts down the mean queue length and reduces the probability of queues of length two or more. Fig 4.3 shows how the waiting time varies with intensity at three representative levels of irregularity in the input; in the limit, for intensities approaching unity, waiting time and queue length tend to the (M/M/1) system.

5.4 <u>Further work and applications</u>

It would be interesting to develop the work on connection type phenomenon to solve the queueing model for the signalling directly rather than use an approximation. However the most important aspect of the work is the case for a stochastic view of railway operations especially in the case of urban passenger operations where a better understanding of station capacity is needed. Finally the steady state approach to the problem is somewhat limited, it would be useful to know how quickly the steady state is reached, that is a full time dependent solution to the problem from which it would be possible to calculate the time needed to recover from a long queue. (Relaxation time) A correlation study of arrival and service processes would complete the queueing approach to signalling.

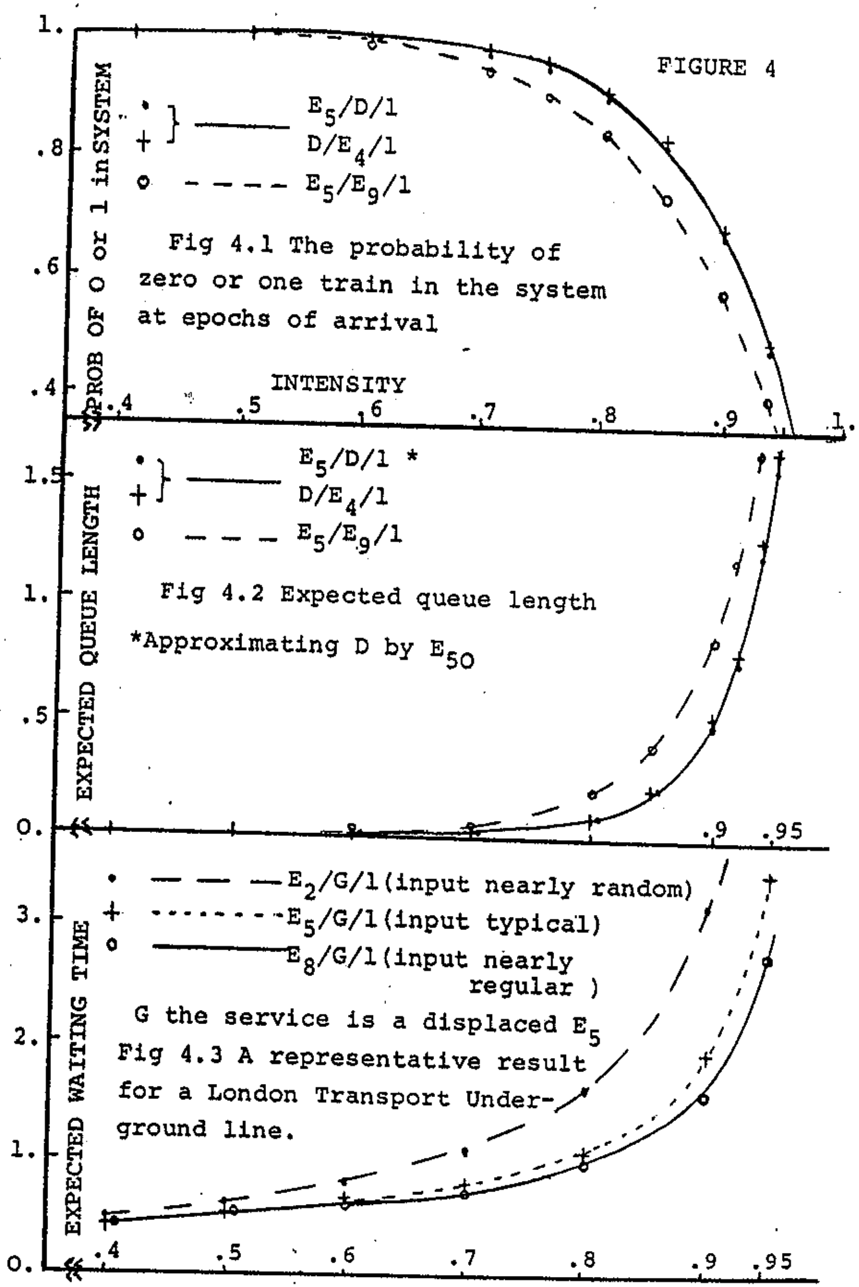


Fig 4.1 The probability of zero or one train in the system at epochs of arrival

Fig 4.2 Expected queue length

Fig 4.3 A representative result for a London Transport Underground line.

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APPENDIX

<u>Lemma 1</u>

The minimum time to cover a distance (d_3+L), the distance of the outer home signal to platform stop starting from rest is

$$t_o = \{2(a_1^{-1}+f_1^{-1})(d_3+L)\}^{\frac{1}{2}}$$

where a_1 and f_1 are the rates of acceleration and deceleration, and t_o is the minimum time.

<u>Lemma 2</u>

Suppose the train is travelling at a speed u, when distant X from the station stop, then t_o, the +ve root of the quadratic, gives the minimum time to cover X.

$$X = (ut_o+\tfrac{1}{2}a_1t_o^2 - u^2/2f_1)/(1+a_1/f_1)$$

both lemma 1 & 2 assume no coasting.

<u>Lemma 3</u>

It is possible for an arrival at the outer home signal to run into the station earlier than a train stopped at the signal would enter (fig.2, case 3) Consider a train travelling at speed u, braking before the signal, it accelerates the instant the signal clears, then it is possible that y seconds later if:

$$uy \geqslant u^2/2f_1 \qquad \text{it overtakes}$$

Since $u<v$, then this is only true for $y \leqslant v/2f_1$. This assumes no speed differential acceleration rates.

<u>Expressions for delay in the four cases of section 3.3</u>

The modifications to equation (4) are derived in terms of the minimum headway eqn.(3). Define $Q_n=(X_n+U_n-Z_n)$. In the notation of sections 2&3 :-

Case 1 $X_{n+1} = 0$

When Q_n is negative, zero delay as the signal is clear when train arrives.

Case 2 $X_{n+1} = K_1(X_n+U_n-Z_n)^2$

When Q_n is positive, the train brakes for Q_n till the signal is clear and then reaccelerates to speed v before the braking point of the station approach; if it cannot reach speed v, it is considered as case 3.

$$X_{n+1} = \tfrac{1}{2}f_1(X_n+U_n-Z_n)^2(1+f_1/a_1)/v$$

Case 3 $X_{n+1} = (X_n+U_n-Z_n)+K_2$

When $Q_n < v/f_1$, the train is not halted, but cannot approach the station at full speed v. The result follows by application of lemma 2, from which t_o is found.

$$X_{n+1} = X_n+U_n-Z_n + t_o-\{((d_2+d_3+L)/v)+\tfrac{1}{2}v/f_1\}$$

Where the term in brackets is the unchecked run in time (Strictly K_2 is not independent of Q_n through t_o)

Case 4 $X_{n+1} = (X_n+U_n-Z_n)+K_2'$

When $Q_n > v/f_1$, the train is halted at the outer home signal for (Q_n-v/f_1). The result is similar to case 3 and follows by application of lemma 1 from which t_o' is defined (K_2' is here strictly independent of Q_n).

$$X_{n+1} = X_n+U_n-Z_n + t_o'-\{((d_2+d_3+L)/v) + \tfrac{1}{2}v/f_1\}$$

6. REFERENCES

(1) BERGMANN D.R. (1972), 'Generalized expressions for the minimum time interval between consecutive arrivals at an idealized railway station', Transp Res. 6(4), p327.

(2) LANG A.S. & SOBERMAN R.M. (1964), 'Urban Rail Transit: Its Economics and Technology.' M.I.T. Press.

(3) LAGERSHAUSEN H. (1968) "Capacity of different signalling systems." Monthly Bulletin of the International Railway Congress Association: Cybernetics and Electronics on the Railways 5(9) p297

(4) ASKEW J.R. BEARDWOOD J.E. & NEWBY D (1968) 'A preliminary investigation of railway signalling methods by computer simulation". Trans Sci 2(3) p191-212.

(5) RAHIMI A., HAJDU L.P. & MACOMBER H.L. (1973) "Safety analysis for automated transportation systems", Transportation Science, 7(1), p1-17.

(6) MORIMURA H. (1969), "An evaluation method of railroad capacity using a 'Pseudo-diagram' model", Proc. 5th Intn. Conf. on O.R., Tavistock, London. p339-346.

(7) YAMADA A. (1971), "On the interference time duration at the railway intersection", Jnl. of Ops Res of Japan 13, p111-128.

(8) KENDALL D.G. (1951), "Some problems in the theory of queues", Jnl. Roy. Statist. Soc. B 13, p151.

(9) BUCKLEY D.J. (1968), "A Semi-Poisson model of traffic flow", Trans Sci 2(2) p107-133.

(10) DAWSON R.F. (1968), "The Hyperlang probability distribution, a generalized traffic headway model", Fourth Intn. Symp. on Transportation and Traffic Theory, Strassenbau und Strassenverkehrstechnik 86 p30-36.

(11) MILLER A.J. (1961), "A queueing model for road traffic flow", Jnl. Roy. Statist. Soc. B 23(1) p64.

(12) SMITH W.L. (1953), "On the distribution of queueing times", Proc Cambr. Philos. Soc. 49, p449.

(13) WISHART D.M.G. (1959), "A queueing system with service time distribution of the mixed chi-squared type", Ops Res., 7, p174.

(14) KOTIAH T, THOMPSON J. & WAUGH W. (1969) "Use of Erlangian distributions for queueing systems" J.Appl.Prob. 6 p584